Twenty First Annual University of North Georgia
Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. What is the maximum possible <u>area</u> of a rectangle in which its perimeter is equal to its area?

   a) 32

   b) $\frac{1}{4}$

   c) 25

   d) 16

   e) None of the above

2. A sector of a circle has angle à Find the value of àáin radians, for which the ratio of the

   a) 2.3

   b) 2

   c)

3. Find the definite integral: $\int_{2}^{1} \frac{1 \quad x^2}{1 \quad 2^x}\, dx$

a) $3 \quad 2^2$

b) $3 \quad 2^{\ 2}$

c) —

d) $\frac{28}{3}$

e) None of the above

4. Find the limit: $\lim_{x} \ x \quad x^2 \ln \frac{1 \quad x}{x}$

a) $\frac{1}{3}$

b) 0

c) $\frac{1}{2}$

d) $\frac{2}{3}$

e) None of the above

5. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions (length and width) of the rectangular portion of a Norman window of maximum area if the total perimeter is 26 feet.

a) length $\frac{26}{4}$, width $\frac{26}{4}$

b) length $\frac{52}{4}$, width $\frac{26}{4}$

c) length $\frac{52}{8}$, width $\frac{52}{8}$

d) length $\frac{52}{8}$, width $\frac{26}{8}$

e) None of the above

6. Find the definite integral: $\frac{}{2} \atop 0$ $\frac{dx}{\sqrt{\sin x} \quad \sqrt{\cos x}^{\,4}}$.

a) $\frac{1}{7}$

b) $\frac{}{7}$

c) $\frac{1}{3}$

d) $\frac{\sqrt{2}}{5}$

e) None of the above

7. The area between the graph of $y \quad \ln\ 1/x$, the x-axis, and the y-axis is revolved around the x-axis. Find the volume of the solid it generates.

a) $\frac{}{3}$

b)

c) $\frac{2}{3}$

d) 2

e) None of the above

8. Find the limit: $\lim_{n} \quad \frac{n}{n^2} \quad \frac{n}{n^2 \quad 1} \quad \frac{n}{n^2 \quad 4} \quad \cdots \quad \frac{n}{n^2 \quad (n \quad 1)^2}$

a) $\frac{}{4}$

b) $\frac{}{2}$

c) $\frac{3}{4}$

d)

e) None of the above

9. Find the definite integral: $\int_{0}^{\overline{\,4\,}} \sec^3 x \, dx$

a) $0.5 \quad \sqrt{2} \quad \ln 2$

b)

c) $0.5 \quad \sqrt{2} \quad \ln 3$

d) $0.5 \quad \sqrt{2} \quad \ln \quad \sqrt{2} \quad 1$

e) None of the above

10. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the

rate (in meters per second) is?7d)rate6l:?TBT1 0 0 1 351.43 497.34936 [( )]TETBT1 0 0 1/C2ID 34>BDC B

11. The curve $x^4 y \quad 12y \quad 144$ has one maximum point and two points of inflection. Find the area of the triangle formed by the tangents to the curve at these three points. Answers are reported in square units.

a) 1
b) 2
c) $\frac{4}{3}$
d) $\frac{3}{2}$
e) None of the above

12. Find the limit: $\lim_{x} \frac{\ln \; 1 \quad 3^x}{\ln \; 1 \quad 2^x}$

a) $\frac{\ln 9}{\ln 4}$
b) $\frac{3}{2}$
c) 1
d) $\frac{\ln 3}{\ln 2}$
e) None of the above

13. Find the integral:

a) $x \ln x \quad x \quad \frac{1}{2} x \ln x \quad C$
b) $\ln \qquad \frac{1}{2} \; - \; {}^{2x}$
c) $x \ln x \quad x \quad \frac{1}{2} \; e^x \quad e^{2x} \quad x^x \quad C$
d) $x \ln x \quad x \quad \frac{1}{2} \; x \ln x \quad x^x \quad C$
e) None of the above

14. Find the integral: $\frac{\cos^2 x}{1 \quad \sin^2 x} dx$

a) $\sqrt{2} \arctan \; \sqrt{2} \tan x \quad\; x \quad C$

b) $2 \arctan \; 4 \tan x \quad\; x \quad C$

c) $\sqrt{2} \arctan \; 2 \tan x \quad\; x \quad C$

d) $4 \arctan \; 2 \tan x \quad\; x \quad C$

e) None of the above

15. Find the minimum value of the function $y \quad 2x^2 \quad 34x \quad 34 \quad e^{x \quad 34}$.

a) $\frac{18}{e^{34}}$

b) $\frac{26}{e^{36}}$

c) $\frac{18}{e^{28}}$

d) $\frac{34}{e^{34}}$

e) None of the above

16. Order the following six functions from slowest growing to fastest growing as x .

$(i)\; 2^{5x} \quad (ii)\; x^x \quad (iii)\; \ln(7x)^{7x} \quad (iv)\; 1 \quad e^{x/3} \quad (v)\; \sqrt{x^2 \quad 5}$

a) $ii, \; iv, \; i, \; v, \; iii$

b) $i, \; iii, \; v, \; ii, \; iv$

c) $v, \; i, \; iii, \; ii, \; iv$

d) $v, \; i, \; iv, \; ii, \; iii$

e) None of the above

17. Evaluate the integral in terms of natural logs.

$$\int_{1/13}^{3/17} \frac{dx}{x\sqrt{1 \quad 25x^2}}$$

a) $\ln 3$

b) $\frac{1}{5}\ln \frac{3}{17} \quad \frac{1}{5}\ln \frac{1}{13}$

c) $\frac{1}{5}\ln \ 13$

d) $\frac{1}{5} \ \ln \frac{3}{17} \quad \ln \frac{1}{13}$

e) None of the above

18. A worker ant at a super colony makes a long walk to bring food to her colony. She starts out at her mount located at the point $A \ \ln 2, 2\sqrt{2}$ along the curve $r_1(\ ) \quad \sqrt{2}\,e \ ,$ $0 \qquad \ln 2$. At the end of this path, she then walks the straight path r_2 that connects the point $B \ 0, \sqrt{2}$ down to the origin $O(0,0)$. This leads to the final trail $r_3(\ ) \qquad ^2,$ $0 \qquad \sqrt{21}$. Find the total distance walked by this ant if is in radians.

a) $3 \quad 5\sqrt{2}$

b) $\sqrt{\ }$

c) $41 \quad \sqrt{2}$

d) $39 \quad 2\sqrt{2}$

e) None of the above

19. Calculate the n^{th} derivative for $f(x) \quad x^n e^x$ at $x \quad 0$.

a) ———

b) 0

c) $n(n \quad 1)(n \quad 2)\cdots 3 \ 2 \ 1$

d) $n\ln(n \quad e)$

e) None of the above

20. Let f be a real-valued function defined on the interval $1,1$ such that

$\sqrt{\quad}$ for all x in $1,1$ and let f^{1} be the inverse function of f.

Then f^{1} (2)

a) 1

b) $\frac{1}{3}$

c) $\frac{1}{e}$

d) $\frac{1}{2}$

e) None of the above

21. Suppose $f(1)$ $1,$ f (1) $2,$ $g(1)$ $3,$ and g (1) $4.$ Let $h(x)$ $\frac{f(x)}{g(x)}$. Find h (1).

a) $\frac{2}{9}$

b) $\frac{2}{9}$

c) $\frac{1}{9}$

d)

e) None of the above

22. Find the definite integral: $\int_0^1 \; x^{3m} \quad x^{2m} \quad x^m \quad 2x^{2m} \quad 3x^m \quad 6^{\frac{1}{m}} \, dx$.

a) $\dfrac{(11)^{\frac{m \;\; 1}{m}}}{33(m \quad 1)}$

b) $\dfrac{(6)^{\frac{m \;\; 1}{m}}}{11(m \quad 1)}$

c) $\dfrac{(33)^{\frac{m \;\; 1}{m}}}{11(m \quad 1)}$

d) $\dfrac{(11)^{\frac{m \;\; 1}{m}}}{6(m \quad 1)}$

e) None of the above

23. Find the definite integral: $\int_0^{/4} \dfrac{\sin \quad \cos}{\sin \quad \cos \quad \sqrt{\sin \quad \cos \quad \sin^2 \quad \cos^2}} \, d$.

a) 0

b) $\dfrac{}{6}$

c) $\dfrac{}{3}$

d) Does not exist

e) None of the above

24. Find the limit: $\lim_{x \quad 0} \dfrac{\int_{1}^{x^2 \quad 1} \ln t \, dt}{x^3}$.

a)

b)

c) 0

d) 2

e) None of the above

25. Suppose f is a continuous function and $f\ (x)$ exists everywhere. If $f(2) \quad 10$ and $f\ (x) \quad\quad 3$ for all x, then what is the smallest possible value for $f(4)$?

a) 1

b) 2

c) 3

d) 4

e) None of the above

26. Which of the following statements are true?

I. If f is differentiable at a, then it has a limit at a.

II. $\dfrac{x^2 \quad 4}{x \quad 2} \quad x \quad 2$

III. | 25x | is differentiable everywhere.

IV. ___________

a) I and II
b) II and III
c) I and III
d) I and IV
e) None of the above

Reminder

Question 30 will be used as a tie-breaker, if necessary.

30. Write a formula for the second derivative of the composition $f \circ g \; (x)$ using $f, g, f \;, g \;, f \;\;, g \;\;.$

a) $f \;(x) g \;(x)$

b) $f \quad g(x) \quad g\,(x) \;^{2} \quad f \quad g(x) \;\; g\,(x)$

c) $f \quad g(x) \quad g\,(x) \;^{2} \quad f \quad g(x) \;\; g\,(x)$

d) $f \quad g(x) \;\; g\,(x) \quad f \quad g(x) \;\; g\,(x)$

e) None of the above

31. Find the definite integral: $\frac{\overline{\;}}{4} \quad \dfrac{x^2 \sin x \quad 1}{1 \quad 2\cos^2 x}\, dx \quad \frac{\overline{\;}}{4}$

a) $\dfrac{\sqrt{3}}{9}$

b) $\dfrac{\sqrt{3}}{3}$

c) 0

d) $\dfrac{5}{3}$

e) None of the above

32. Find the smallest value of the constant k such that $f(x) \quad kx \quad 1 \quad \dfrac{1}{x} \quad 0$ for all $x \quad 0.$

a) 4

b) $\dfrac{1}{4}$

c) 16

d) $\dfrac{1}{16}$

e) None of the above

33. Find the limit: $\lim_{x} \sqrt{x^{200} \quad x^{100} \quad 1} \quad x^{100}$

a) 3

b) $\frac{1}{2}$

c) 6

d) $\frac{1}{6}$

e) None of the above

34. Find the definite integral: ${}_{0}^{1} \frac{x^4(1 \quad x)^4}{1 \quad x^2}\, dx$.

a)

b) 7

c) 7

d) $\frac{22}{7}$

e) None of the above

35. Let $f(x) \quad \begin{array}{ll} \frac{\cos x}{\frac{}{2} \quad x} & \text{if } x \quad \frac{}{2} \\ a^2 \quad a \quad 1 & \text{if } x \quad \frac{}{2} \end{array}$.

For which value of a is the function f continuous at $\frac{}{2}$?

a) $a \quad 1$

b) $a \quad 2$

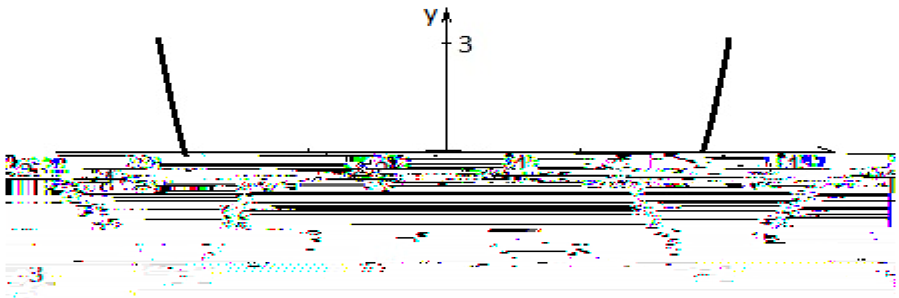
c) $a \quad 0$ and $a \quad 1$

d) $a \quad 2$

e) None of the above

36. Identify the graph of $k(x) \quad x^{2/3} \quad x^2 \quad 4$.

a)



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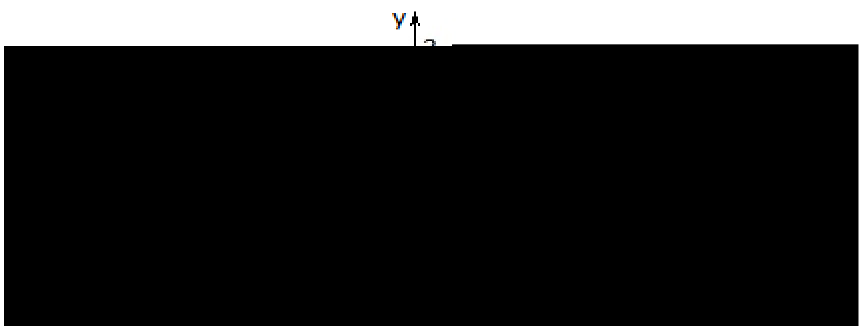

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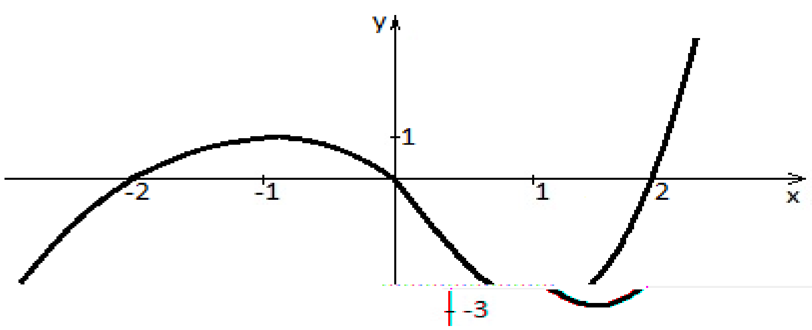
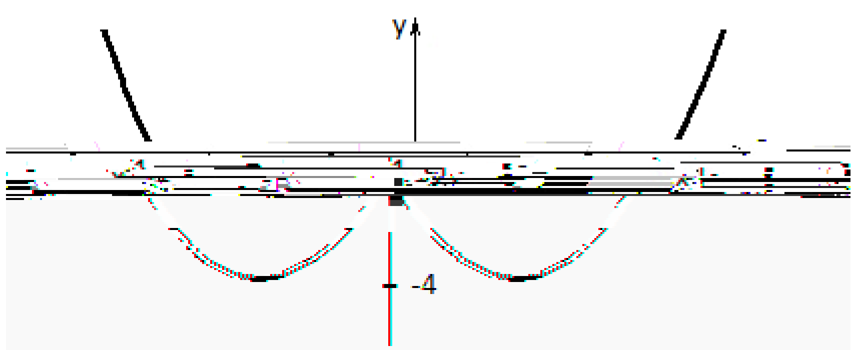


d)



e) None of the above

37. Find the maximum value of the function $f(x) \quad \frac{\sin x \quad 1}{\sin^2 x \quad \sin x \quad 1}$.

a) The maximum value of f is 0 .
b) The maximum value of f is 1.
c) The maximum value of f is 4.
d) The maximum value of f does not exist.
e) None of the above

38. A cylindrical tank 10 feet high with radius 6 feet is full of water. Set up an integral to find the work required to empty the tank. The water weight is 62.4 lb/ft^3.

a) $\displaystyle {}_{0}^{10}\; 62.4 \quad 6^2 x \, dx$, where x is the distance to the top
b) $\displaystyle {}_{0}^{10}\; 62.4 \quad 6^2 (10 \quad x) \, dx$, where x is the distance to the bottom
c) Both a) and b) are correct
d) Only a) is correct and b) is incorrect
e) None of the above

39. Find the area enclosed between the line $y \quad 2x$ and the parabola $y \quad x^2$.

a) $\frac{3}{4}$
b) $\frac{4}{5}$
c) $\frac{4}{3}$
d) $\frac{5}{3}$
e) None of the above

40. If $A \quad {}_{0}^{/2} \dfrac{\sin^n x}{\sin^n x \quad \cos^n x} dx$ for any $n \quad 0$, then

a) $A \quad \dfrac{}{2}$

b) $A \quad \dfrac{}{4}$