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Gainesville State College

Fifteenth Annual Mathematics Tournament

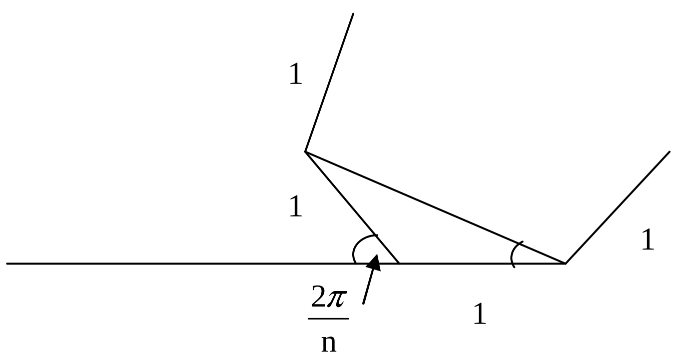
April 4, 2009

Solutions for the Afternoon Team Competition

Round 1

There are 6 terms that repeat in the sequence (5, 0, 0, 0, -5, -5). When you divide 3997 by 6, the quotient is 666 with a remainder of 1. So the 1^{st} term in the sequence is 5.

Round 2



For a regular n-sided polygon, the external angle is $\frac{2\pi}{n}$.

The shortest diagonal will be (as seen in figure) obtained if a triangle is formed.

Thus, the shortest diagonal is the base of an isosceles triangle with external angle $\frac{2\pi}{n}$.

The base angles must be $\frac{1}{2}\left(\frac{2\pi}{n}\right) = \frac{\pi}{n}$.

Therefore, the base is $2\cos\frac{\pi}{n}$.

Round 3

On each side there are 19 x 19 small cubes with exactly one side painted, so the total of these is $6 \text{ x } 19 \text{ x } 19 \; = \; 2{,}166$.

Then, on each edge we have 19 cubes that have two sides painted for a total of $12 \text{ x } 19 \; = \; 228$ cubes with two sides painted.

Finally, on each corner we have a cube with three sides painted for a total of 8 of these.

The total is $2{,}166 + 228 + 8 \; = \; 2{,}402$.

Round 4

$\theta \quad \frac{6}{16}$

Round 8

From the diagram, vertex E must lie on the line $y = x$ and $y = -\frac{b}{a}x + b$ (as a point on $y = -\frac{b}{a}x + b$). So it has coordinates $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$ found by solving these equations simultaneously.

By symmetry, we must have coordinates for vertex F as $\left(\frac{a^2}{a+b}, \frac{b^2}{a+b}\right)$.

Applying the distance formula, we get

$$d(EF) = \frac{\sqrt{a^2(a-b)^2 + b^2(b-a)^2}}{a+b} \qquad \frac{\sqrt{(\quad)^2(\quad)}}{\quad}$$