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dede.delaughter@ung.edu or call 706-310-6207.*

Math Necessities

for Review and/or Instruction

for

- Learning Support
- College Algebra
- Mathematical Models
- Quantitative Skills
- SAT/ACT review
- GACE review

Rules for Operations with Integers

Integers are whole numbers, positive or negative.

Operations with integers are the same as operations with whole numbers.

Example: $12 - 2 + 8 = 10$

1. Add or subtract from right to left.

2. If there is a minus sign, change the sign of each term after it.

3. Then add or subtract from right to left.

$$\text{C. } -8 + 3 = -5 \quad \text{(Subtract)}$$

To add a positive and a negative, move rightward and take the sign of the larger number.

OR DIRECTLY: $7 - 2 = 7 + (-2) = 5$

D. Subtraction: To subtract a #, you add its opposite. (Rewrite if needed.)

(After rewriting, use your rules for addition.)

$$\begin{array}{r} 7 - 2 \quad \text{OR} \quad 7 - 2 \\ 5 \qquad \qquad 7 + -2 \\ \qquad \qquad \qquad 5 \qquad \qquad 8 + +7 \\ \qquad \qquad \qquad \qquad \qquad 15 \end{array}$$

II. Multiplication and Division

B. $3 \times -2 = -6$ Positive \times Negative = Negative

why? 3×-2 means $-2 + -2 + -2 = -6$

C. $-2 \times 5 = -10$ Negative \times Positive = Negative

$$\begin{aligned} &(-)(2)(-)(5) \\ &(-)(-)(2)(5) \\ &(+)(10) \\ &(10) \end{aligned}$$

General Guidelines for Solving Equations

Name _____

1. ~~Isolate the variable~~ Get the variable by itself on one side of the equation with only a coefficient.

2. ~~Simplify~~ Clear parentheses and decimals.

Fractions: by multiplying both sides of the equation by the LCD

Decimals: by multiplying both sides of the equation by a power of 10

terms)

Like terms: terms that have the same numerical coefficient for their dimensional

factors. If we add or subtract like terms with exactly the same

exponents

If we add or subtract like terms, the only thing that changes is

the numerical coefficient (the number out front).

2. Add or subtract to get all the variables (letters) on one side

other side

3. Multiply or divide to get the coefficient of the variable to be one

(multiplication or division)

4. If there is a fraction, multiply by the reciprocal

Exponents

Name _____

I. When you multiply like bases you add exponents

II. When you divide like bases you subtract exponents

$$\frac{20x^7 p^3}{35x^5 p^8} = \frac{4x^2}{7p^5}$$

III. When you multiply or divide terms with different variables

IV. When combining (adding or subtracting)

you can only combine (add or subtract) terms that have exactly the same variables with exactly the same exponents

$$13a^4 c^5 + 5a^4 c^2 + 4a^4 c^5 = 17a^4 c^5 + 5a^4 c^2$$

V. Multiplying Polynomials

A. Only for a binomial x a binomial

1. FOIL may always be used. This is just a method that helps

make sure each term in the first parenthetical is multiplied by each term in the second parenthetical.

(unless your terms in the second parenthetical are the same as in the first parenthetical). O (Outermost terms), I (Innermost terms), L (Last terms in each parenthesis).

1. (Last terms in each parenthesis). Then combine like terms.

2. Special product patterns. (Note: FOIL or distributive could always be used on any of these, but the next chapter will make it easier if you learn the patterns.)

$$(a+b)(a-b)$$

$$(a+b)^2$$

$$(a-b)^2$$

$$1. \sqrt{49}$$

$$2. \sqrt{98}$$

$$3. \sqrt{25}$$

$$4. \sqrt{75}$$

$$5. \sqrt{100}$$

$$6. \sqrt{900}$$

$$7. \sqrt{700}$$

$$8. \sqrt{400}$$

$$9. \sqrt{4}$$

$$10. \sqrt{12}$$

$$11. \sqrt{16}$$

$$12. \sqrt{32}$$

$$13. \sqrt{48}$$

$$19. \frac{3+\sqrt{49}}{2}$$

$$20. \frac{3-\sqrt{49}}{2}$$

$$21. \frac{8+\sqrt{36}}{2}$$

$$22. \frac{8-\sqrt{36}}{2}$$

$$23. \frac{12+\sqrt{16}}{4}$$

$$24. \frac{12-\sqrt{16}}{4}$$

$$25. \frac{30+/-\sqrt{64}}{2}$$

Transformations on Parabolas and Absolute Value Name _____

Parabola

$$y = x^2$$

$$\begin{array}{c} y = x^2 \\ \text{opens up} \end{array}$$



$$\begin{array}{c} y = -x^2 \\ \text{opens down} \end{array}$$



Absolute Value

$$y = |x|$$

$$\begin{array}{c} y = |x| \\ \text{opens up} \end{array}$$



$$\begin{array}{c} y = -|x| \\ \text{opens down} \end{array}$$



The number in front of the basic function affects how wide or narrow the graph is.

2

$$y = 3(x - 2)^2 + 4$$

shifts: $\rightarrow 2$ $\uparrow 4$

$$y = |x + 5| - 2$$

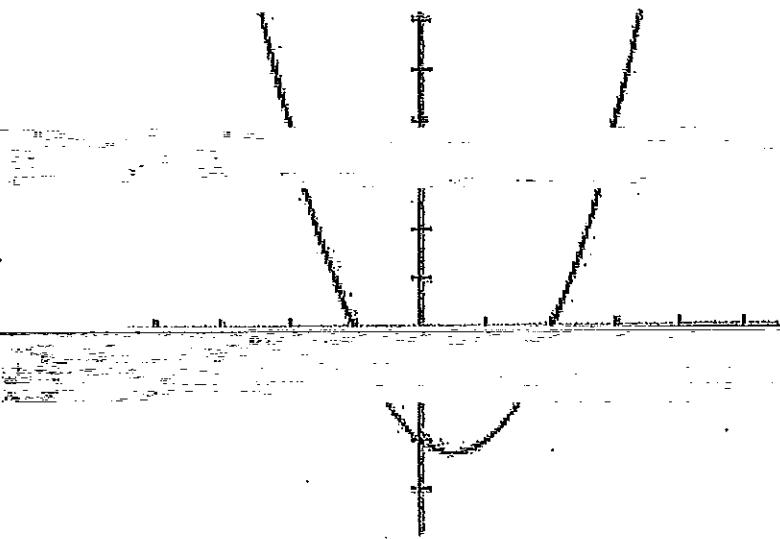
shifts: $\leftarrow 5$ $\downarrow 2$

A #

A number in front of the basic function shifts the graph in the opposite sign way.

From: $y = a(x - h)^2 + k$ the vertex is (h, k) .

2a



This parabola is a example of the graph of the equation:

$$y = x^2 - x - 2$$

The zeros are where _____ = zero.

How many zeros are on this parabola?

What are the zeros (from the graph)?

Two algebraic methods that can be used to find the zeros are:

I. Factoring

II. Quadratic Formula

Factoring Methods

Always look for GCF first.

$$(a+b)(a-b)$$

$$(a+b)(a-b)$$

4 terms by grouping

difference of two squares

4 terms
Try a. when you have 3 perfect squares

4. Factor by grouping

a. 1 and 3 or 3 and 1

b. 2 and 2 (involves factoring out GCFs)

3 terms

6. Trial and error

factors as

$$(a+b)(a^2 - ab + b^2) \quad (a-b)(a^2 + ab + b^2)$$

Solving Quadratic Equations

Name _____

Methods of Factoring

of the equation by itself.
Take the square root of both sides.

2. Difference of 2 squares

3. Completing the square:

Given: $x^2 + bx = \#$
Add $(\frac{b}{2})^2$ to both sides to obtain a perfect square on the left.

4. Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Equation

a =

b =

c =

2a

Discriminant!

Name _____

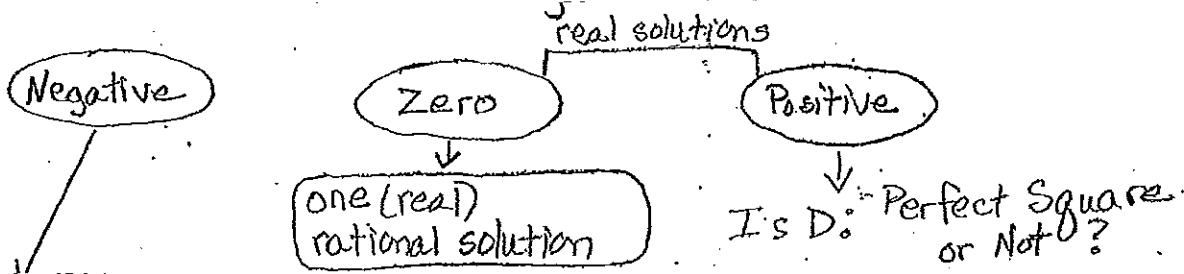
With Practice

Quadratic Equation

a = b = c =

$$D = b^2 - 4ac$$

To D: Negative, or Zero or Positive?



Graphing Lines

Name: _____

Main methods for graphing lines:

- Realizing that the equation of the line may tell us that all of the points on the line may have the same x-coordinate (ex: $x = 2$) or the same y-coordinate (ex: $y = -3$).

Ex. $y = -4$ (Note: I chose #'s for x)

(Note: I chose #'s for x)

-8 | -4

X

different points

run

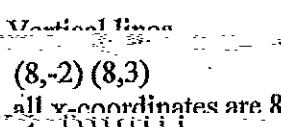
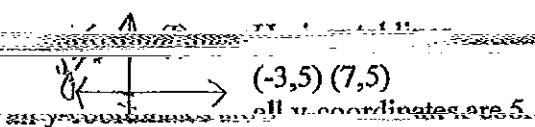
$$m = \frac{3}{2} \text{ up 3 or down 3}$$

right 2 left 2

Substitute 0 in for y & solve for the x value. Plot the point.

Writing Equations of Lines

Name _____



↓ 8(i) Using $y = mx + b$ (slope-intercept form)

(A) Given m & b, just substitute in.

(B) Given two points $(-3, 4), (7, -10)$

The slope

and one

point

Start here

$$-10 = -7/5(7) + b$$

(3) Solve for the value of b

$$-10 = -49/5 + b$$

$$-50/5 + 49/5 = b$$

$$-1/5 = b$$

(4) Substitute just one point in for x, 1, into the equation. $x = 1$ to stand for all

$$y = -7/5x - 1/5$$

* Note: we can choose any coordinate b in the

y coordinate in that point (i.e.) $(0, 5) \quad b=5$

(III) Using $y - y_1 = m(x - x_1)$ (point-slope form)

(A) Given the slope and one point: just substitute the point in for (x_1, y_1) & the slope in for m, then simplify to get in the desired form.

Ex: $(3, -5)$ $m = 7$, put the answer in slope-intercept form

$$y - 5 = 7(x - 3)$$

$$y + 5 = 7x - 21$$

$$y = 7x - 26$$

(B) Given two points: $(7, 4), (0, -9)$

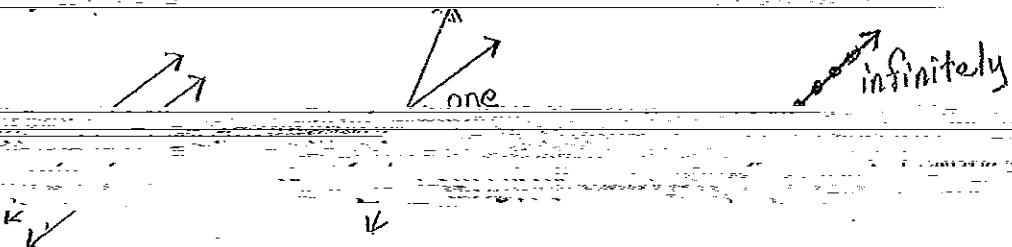
$$2/15 \cdot x + b = 16/15 \quad \text{ok, but below would look better:}$$

$$2/15 \cdot x - 16/15 \quad \text{(multiplied both sides by 15 to get this)}$$

I. Graphing Method

1. Graph one of the lines. Using $y = mx + b$ may be helpful.

Possible types of solutions:



II. Substitution Method

1. Solve for one or more variables in one or more equations. In one of the equations, let $x =$ or $y =$.

4. Substitute that new value (into either equation) to solve for the other variable.

5. Write your ordered pair solution.

III. Addition (Elimination) Method

1. Multiply one or both of the equations by a number to obtain opposite.

5. Write your ordered pair solution.

Solving Systems with Matrices/ Determinants Name _____

Solving:

Solve:

$$\begin{aligned}x + 4y - z &= 10 \\2x + 5y - 3z &= 7\end{aligned}$$

$$8x + y - 2z = 11$$

Write the matrix equation:

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & x \\ 2 & 5 & -3 & y \\ 8 & 1 & -2 & z \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -1 & 10 \\ 2 & 5 & -3 & 7 \\ 8 & 1 & -2 & 11 \end{array} \right]$$

The process overall:

$$AX = B$$

$$X = A^{-1}B$$

Note: Matrix mult. is
NOT commutative;
 BA^{-1} will NOT give you

Enter matrix A and matrix B into TI-83 plus

II. Solve by multiplying A^{-1} times B

To find inverses:

2nd Mode (Quit)

2nd Mode (Quit)
2nd Matrix 1 [x]
enter
To get in fraction form:

3
4

Determinants:

To find the determinant:

Enter matrix A as above, then:

2nd Mode (Quit)

2nd Matrix 1

2nd Matrix 1 close parentheses enter

End Behavior (using Highest Degree)

Name

Highest
Degree

EVEN

ODD

and Remember:

End Behavior If LC is If LC is If LC is If LC is

(Up the graph positive, negative, up positive, negative)

and right ends)

The end behavior is

The end behavior is

The end behavior is

log means exponent

$\log = \text{exponent}$

base 10

means: what exponent do you raise 2 to, to get 8?

If no base is written, then the base is understood to be 10.

$\log 100$ "What exponent do you raise 10 to, to get 100?" - 2

$\log 10^x$ "what exponent do you raise 10 to, to get 10^x ?"

$e = 2.718\dots$

$\ln e$ "natural log"

$\ln e$ "What exponent do you raise e to, to get e?" 1

$\ln 10$ "What exponent do you raise e to, to get 10?"

EXPONENT FACTS

A common sense phrase to remember.

$1^2 = 1$	$1^3 = 1$	$(-1)^2 = -1$	$(-1)^3 = -1$
$3^2 = 9$	$3^3 = 27$	$(-3)^2 = -27$	$-3^3 = -27$
$4^2 = 16$	$4^3 = 64$	$(-4)^2 = -64$	$-4^3 = -64$
$5^2 = 25$	$5^3 = 125$	$(-5)^2 = -125$	$-5^3 = -125$
$6^2 = 36$	$6^3 = 216$	$(-6)^2 = -216$	$-6^3 = -216$

$9^2 = 81$	$(-1)^2 = 1$	$-1^2 = -1$
$10^2 = 100$	$(-2)^2 = 4$	$-2^2 = -4$
$11^2 = 121$	$(-3)^2 = 9$	$-3^2 = -9$
$12^2 = 144$	$(-4)^2 = 16$	$-4^2 = -16$
$13^2 = 169$	$(-5)^2 = 25$	$-5^2 = -25$
$14^2 = 196$	$(-6)^2 = 36$	$-6^2 = -36$
$15^2 = 225$	$1^4 = 1$	$1^5 = 1$
$16^2 = 256$	$2^4 = 16$	$2^5 = 32$
$17^2 = 289$	$3^4 = 81$	$3^5 = 243$
$18^2 = 324$	$10^4 = 10000$	$10^5 = 100000$
$19^2 = 361$		
$20^2 = 400$		
$25^2 = 625$		